A RIGOROUS PROOF OF DETERMINISM DERIVED FROM THE SPECIAL THEORY OF RELATIVITY

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A proof is given that there does not exist an event, that is not already in the past for some possible distant observer at the (our) moment that the latter is "now" for us. Such event is as "legally" past for that distant observer as is the moment five minutes ago on the sun for us (irrespective of the circumstance that the light of the sun cannot reach us in a period of five minutes). Only an extreme positivism: "that which cannot yet be observed does not yet exist", can possibly withstand the conclusion concerned. Therefore, there is determinism, also in micro-physics.

Consider an inertial system $(X_1O_1T_1)$ with the observer W_1 in O_1, as well as the inertial system $(X_2O_2T_2)$, observer W_2 being in O_2. Consider both W_1 and W_2 at the time $T_1 = 0$. $(X_2O_2T_2)$ moves at a constant velocity in the direction of O_1. X_2 intersects T_1 in P. Then we find that for W_1, observer W_2 in O_2, in the drawn position (event B), is "now". For W_2 in B, however, events B and P occur simultaneously: for W_2 event P is "present" at the same moment $T_1 = 0$ of W_1 that W_2 is "present" for W_1. (In the coordinate system of W_2, event A occurs simultaneously with his own experience V.)

This "now" P for W_2—in the "absolute future" of W_1—is as real as, e.g., W_2 is for W_1. Conclusion: P is completely determined at the moment $T_1 = 0$ of observer W_1. P_1 is even "past" already for W_2. With each event P_1 in the future of W_1 we can now think of an observer W_2 for whom, at the moment of observer W_1 that W_2 is "now" for W_1, P_1 is already in the past. Thus each event is determined: it is already "past" for some one in our "now". (Signals predicting the future for W_1 would, of course, always be too late, because of the limiting effect of the velocity of light, c.)

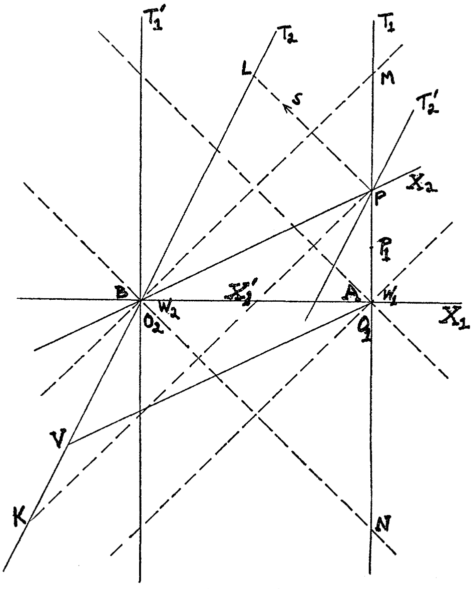

Figure 1

To be completely rigorous, we have to take into account the fact that, strictly speaking, an event is not a point in the four-dimensional continuum, but has a certain temporal (and spatial) extension. For such "real" events, our reasoning is only rigorous if we speak about P_1 and not if we speak about P, since the latter cannot be wholly "present" for W_2 (because of its temporal extension). However, since an "extended" P_1 can always be made wholly "past" for W_2 (also when it is wholly "future" for W_1), if we make AB in figure 1 and, therefore, AP, long enough, this problem offers no

Philosophy of Science 1966.33:341-344.

difficulty with regard to our proof: we can substitute P by P_1 if its temporal duration cannot be neglected. (If we make AB longer, P becomes our "new" P_1.)

It is even completely "legal" when, at the moment that W_2 is in W_1's $T_1 = 0$, that there be a light signal S, departing from P, and which for him (W_2) in the present, begins to move towards W_2, and reaches him later. (E.g., P is, as an event, the emission of the light signal.) If W_2 knows at every moment his distance to W_1 and his velocity with respect to him, he can, after having observed S, compute the location of P in the four-dimensional continuum. Then the whole situation of figure 1 is clear to him, including the fact that P occurred in W_1's future at the moment that W_2 and W_1 were in the places shown in the drawing.

It is even possible that W_2, on experiencing event B, will reduce his velocity until he is at rest with respect to W_1 during a very short time t, so that during this time, his coordinate system is inertial system ($X'_1O'_2T'_1$) (supposing he knows his velocity towards W_1; however, this is not of fundamental interest). Thereafter, W_2 can resume his former velocity. *When, after some time, W_2 receives the light signal S, he will know that already—during the short time t—when he was at rest with respect to W_1, experiencing the same "present" as did W_1 in virtually the same inertial system, W_1 could do nothing at all to prevent event P in his absolute future.* Event P, which a moment before and then unknown to W_2, was "now" (or, if one prefers, "past") in W_2's former (and later) system of coordinates ($X_2O_2T_2$), as will appear later (viz. after he receives S). This implies that P is predetermined, in the sense of the definition to be given below.

After this, we can, in an understandable way, give a rigorous and objective ("absolutistic") definition of the concept "determined" and, therefore, of "determinism":

We say that an event P is (pre-)determined if, for any possible observer W_1 (that is, for all possible observers, and even for all other, e.g., physical, instances), who has P in his absolute future (that is, that the future part of W_1's course through the four-dimensional continuum may eventually pass through P), we can think of a possible observer W_2 (or: there may exist an observer W_2) who can prove, at a certain moment T_p, that W_1 could not possibly have influenced event P in an arbitrary way (e.g., have prevented P) at any moment when P still was future, or was present, for W_1, supposed that W_1 did desire to do so.

This definition is completely independent of the inertial system used and, moreover, it certainly is not less strong logically than the popular notion about something being "predetermined".

To see that P (*or, if one prefers,* P_1) in figure 1 is pre-determined in the sense of the definition, we only have to look at the paragraph above, where W_2, for a short time t, made his velocity equal to W_1's. For, whatever possible observer (with P in his future) W_1 might be, W_2 can make his velocity equal to W_1's for the time t and do the other things spoken of in the paragraph, and the argument of the paragraph applies. (It is easy to see that it even applies if the future part of W_1's course through the four-dimensional continuum will not, eventually, pass through P, P being absolute future to him as before. It is only necessary that the position of P in the four-dimensional continuum can be located afterwards.)

Nor does anything change if W_1 is a physical instance that might influence P.

We now see too that, being "past" (or "present") *for only one* inertial system ($X_2O_2T_2$), an event can be shown to be determined in all other systems, that is, (pre-)determined in the absolute sense of the definition in those systems where P is still in the future.

In figure 1, the moment T_p of the definition will occur, e.g., when W_2 passes event L, or at the point of the intersection of the axes T_1 and T_2, where W_1 can give W_2 all possible information if necessary.

More generally, the signal S in figure 1 may be a photograph of any event P in W_1's future (P not being S's emission now).

By taking the distance AB arbitrarily long we can make AP correspond with an arbitrarily long time, that is, P is pre-determined "from time immemorial".

We emphasize that the proof given above is totally independent of the question whether the events P and P_1 are of a physical or a psychical nature. This excludes the possibility of saving "freedom of will", e.g., by saying that the proof is a physical one, being only applicable to physical processes.

Our conclusion is: there does not exist an event, that is not already in the past for some possible distant observer, at the (our) moment that the latter is "now" for us. Such event is as legitimately past for that distant observer as is the moment five minutes ago on the sun for us (irrespective of the circumstance that the light of the sun cannot reach us in a period of five minutes). Only an extreme positivism: "that which cannot yet be observed does not yet exist", can possibly withstand the conclusion concerned. Therefore, if we take into account the definition and the argument given above, we may conclude that there is determinism (which is, of course, not the same thing as causality), also in micro-physics. Also there is no free will; from this it follows, e.g., that the whole philosophy of existentialism is untenable. In the same way, of course, we can conclude that theories, intending to save free will with the aid of "indeterminism" in micro-physics, cannot be tenable.

One might object—and, I think, among others Professor Grünbaum would object—that all point-events between M and N of figure 1 are topologically simultaneous with event B, and that, therefore, it makes no sense for observer W_2 to say that P_1, e.g., is in the past or in the future. According to Professor Grünbaum, this may only be so in the "metrical" sense. It is, he says, only for the convenience of description that we make the *convention* of "metrical" simultaneity. But this should have no real physical sense (See [2], pp. 349-51).

I cannot accept Professor Grünbaum's interpretation which, in fact, denies both the influence of an observer's velocity on his interpretation of simultaneity and any real physical sense of making a three-dimensional hyperplane-intersection of the four-dimensional space-time continuum correspond with the place and velocity of the observer.

In fact, on p. 354 of his book, Professor Grünbaum admits that only for the special choice of "metrical" simultaneity as, e.g., made in this paper, the velocity of light in the direction AB is equal to the velocity of light in the direction BA.[1] To this "supposition," therefore, the theorem asserting determinism given above applies. Accepting, as isotropy of space requires and as most physicists do, the constancy of light-velocity, this theorem could mean, I think, the definite end of indeterminism, at least in a form in which it is still being favored by many, including both physicists and non-physicists. (See, e.g., references [4], p. 295, [3], pp. 154-57, [1], p. 52, and the works of many quantum-theorists.)

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[1] It is of crucial importance to realize that, if we don't agree that, in W_1's inertial system (see figure 1), B is the only event on T'_1 occurring simultaneously with A—simultaneously in a real physical sense—, *it would not make any physical sense to say that, in* W_1*'s inertial system, the velocities of the light-signals (event* $N \rightarrow$ *event* B*) and (event* $B \rightarrow$ *event* M*) are equal.* Nor would it make any physical sense to speak of a constant velocity of light in W_2's inertial system if we denied that, for W_2, P is *the only event* of T_1 occurring simultaneously with B. For, in that case, it cannot make any physical sense to say that the light-signals (event $K \rightarrow$ event P) and (event $P \rightarrow$ event L) have the same velocity, c, in $(X_2O_2T_2)$.

REFERENCES

[1] Eddington, A. S., Space, Time and Gravitation.
[2] Grünbaum, A., Philosophical Problems of Space and Time, 1964.
[3] Reichenbach, H., Les Fondements *Logiques* de la Mécanique des Quanta.
[4] Whitrow, G. J., The Natural Philosophy of Time, 1961.

Philosophy of Science 1966.33:341-344.